Examples 7 in Basic Geometry

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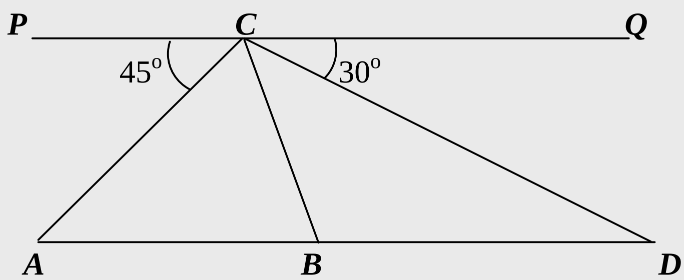
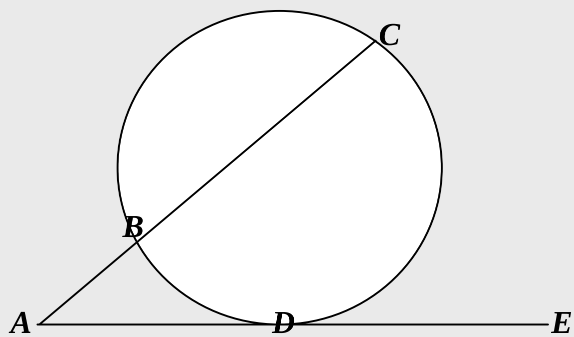
Examples 7

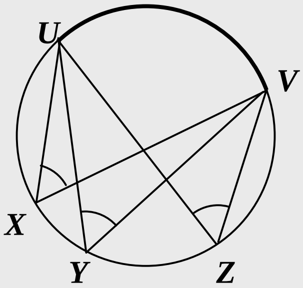
0. Assuming $\angle CBD = \angle ACD$, PQ is parallel to AD, $BD = 5$, and $AD = 8$, find the angle ABC and the length of CD.



1. Assuming $AB = 3$, $BC = 8$, and AE is tangent to the circle, find the length of AD.

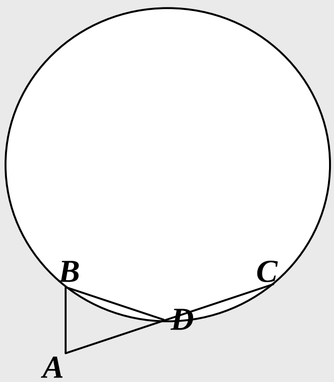


2. We have a fact that in the figure below, the three angles ***X***, ***Y***, and ***Z*** are the same no matter where the three points may be in the circle if they are outside the arc ***UV***.



And if the arc ***UV*** is a half circle, all the angles ***X***, ***Y***, and ***Z*** are 90°.

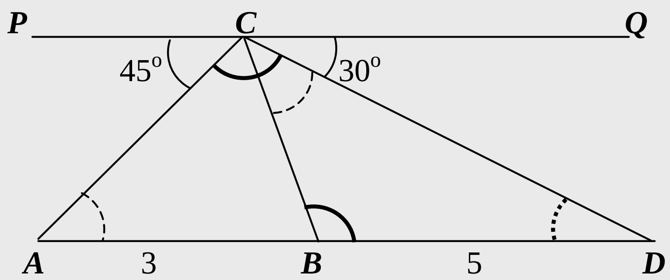
Using the fact above, and assuming in the figure below, ***BD*** = ***DC*** = ***AD***, ***AB*** = **3**, and the diameter of the circle is 8, find the length of ***AD***.



Suggestions or Solutions
To the Problem 0

Assuming $\angle CBD = \angle ACD$, PQ is parallel to AD, $BD = 5$, and $AD = 8$, find the angle ABC and the length of CD.

Fig. 0.0



So putting in the figure above, the angles given and the sides given, we can see that ΔACD is similar to ΔCBD. How then can we find the angle ABC?

First, we can say that $\angle CBD + \angle ABC = 180^o$.

So finding $\angle CBD$, we can get $\angle ABC$.

How then can we find $\angle CBD$?

We know $\angle ACD = \angle CBD$, and in the figure above, can see this:

$45^o + \angle ACD + 30^o = 180^o$.

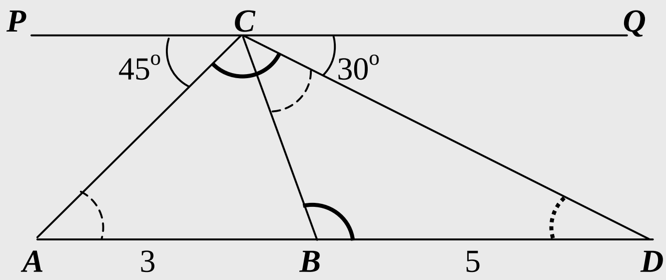
So we get $\angle ACD = 105^o$, and thus, we get $\angle CBD = 105^o$.

So we get $\angle CBD + \angle ABC = 180^o \Rightarrow \angle ABC = 180^o - \angle CBD = 180^o - 105^o = 75^o$.

How then can we find the length of CD?

We know that ΔACD is similar to ΔCBD.

Fig. 0.1



So we can say that AD corresponds to CD, AC corresponds to CB, and CD corresponds to BD. Thus, we get $\frac{AD}{CD} = \frac{AC}{CB} = \frac{CD}{BD}$.

And we have $AD = 8$, and $BD = 5$.

So we get $\frac{AD}{CD} = \frac{CD}{BD} \Rightarrow \frac{8}{CD} = \frac{CD}{5}$.

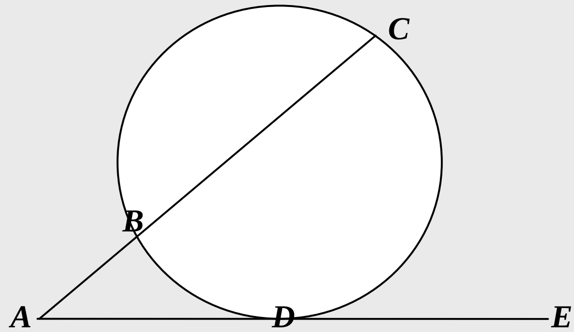
Thus, assuming $c = CD$, we get $c^2 = 40 \Rightarrow c = \pm\sqrt{40} = \pm 2\sqrt{10}$.

And we have $c = CD > 0$. So we get $CD = 2\sqrt{10}$.

Suggestions or Solutions
To the Problem 1

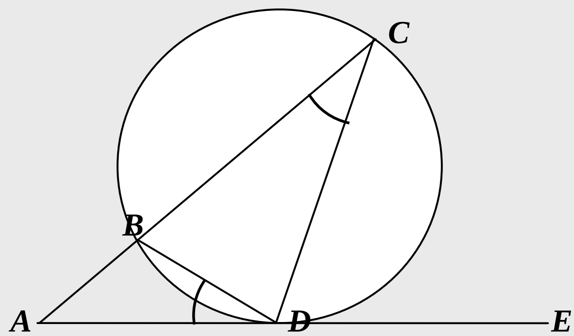
Assuming ***AB*** = **3,** ***BC*** = **8,** and ***AE*** is tangent to the circle, find the length of ***AD*.**

Fig. 1.0



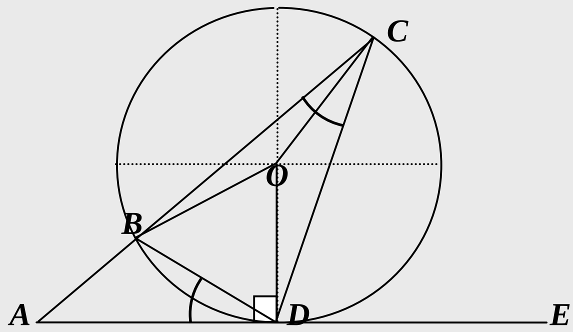
Connecting ***B*** and ***D***, and ***C*** and ***D*** the way below, we can say **Δ*ABD*** is similar to **Δ*ADC***.

Fig. 1.1



We can say that ∠***A*** belongs to both triangles, and ∠***C*** is the same as ∠***ADB***. How are they the same though?

Fig. 1.2



Suppose that the center of the circle is O.
Then first, assuming r is the radius, we can say that $OB = OC = OD = r$.

So next, we can say that $\angle OCB = \angle OBC$, $\angle OBD = \angle ODB$, and $\angle OCD = \angle ODC$.

And in a triangle, the three angles add up to 180^o. So we can see this:

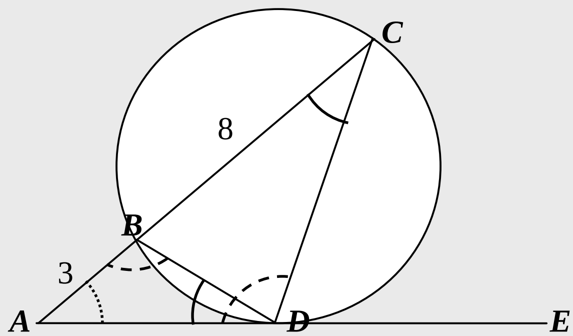
$$\angle OCB + \angle OBC + \angle OBD + \angle ODB + \angle OCD + \angle ODC$$
$$= 2(\angle OCB + \angle ODB + \angle OCD) = 180^o.$$

Thus, we get $\angle OCB + \angle ODB + \angle OCD = 90^o$.

And we know this: $\angle OCB + \angle OCD = \angle C$. So we get $\angle C + \angle ODB = 90^o$.

And we have $\angle ADB + \angle ODB = 90^o$, too. So we get $\angle ADB = \angle C$.

Fig. 1.3



So we can say that ΔABD is similar to ΔADC.

Thus, we get $\frac{AB}{AD} = \frac{AD}{AC}$.

And we have $AB = 3$, and $BC = 8$.

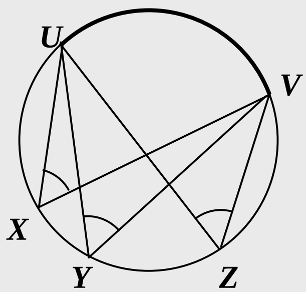
So we get $\frac{3}{AD} = \frac{AD}{11}$.

Thus, assuming $a = AD$, we get $a^2 = 33 \Rightarrow a = \pm\sqrt{33}$.

And we have $a = AD > 0$. So we get $AD = \sqrt{33}$.

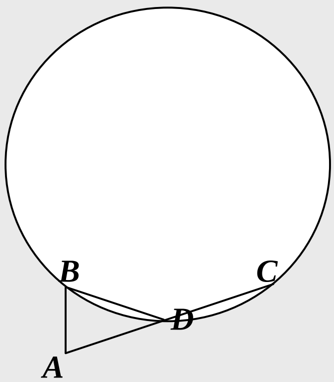
Suggestions or Solutions
To the Problem 2

We have a fact that in the figure below, the three angles ***X***, ***Y***, and ***Z*** are the same no matter where the three points may be in the circle if they are outside the arc ***UV***.



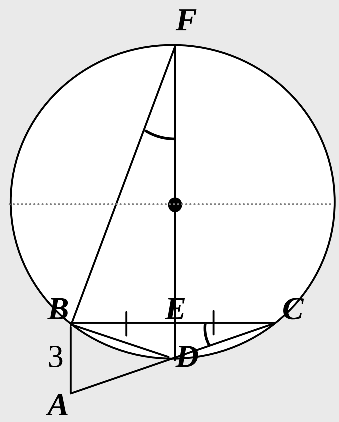
And if the arc ***UV*** is a half circle, all the angles ***X***, ***Y***, and ***Z*** are 90^o.

Using the fact above, and assuming in the figure below, ***BD*** = ***DC*** = ***AD***, ***AB*** = **3**, and the diameter of the circle is 8, find the length of ***AD***.



Let ***FD*** pass though the center of the circle, and ***E*** be the midpoint between ***B*** and ***C***. Then, we can put them the way below.

Fig. 2.0



Then first, we can say that ***FD*** is the diameter.

So using the fact above, we can see that $\angle DBF = 90^{o}$.

Also, using the fact above, we can say that $\angle BCD = \angle BFD$, because both cover the same arc ***BD***.

Next, we have $BD = DC = AD$.

So we can see two isosceles triangles, which are ΔBCD and ΔABD.

Also, we can say that $\angle ABC = 90^{o}$. That is, ΔABC is a right triangle.   Why?

To begin with, in ΔABD, we have $\angle ABD = \angle DAB$, since the triangle is isosceles. And in ΔBCD, we have $\angle DBC = \angle DCB$.

Next, we know in a triangle, the sum of all its angles is 180^{o}.

So taking the sum of all the angles in ΔABC, we can put it the way below.

$2(\angle ABD + \angle DBC) = 180^o$.

Thus, we get $\angle ABD + \angle DBC = 90^o$.

And we know $\angle ABC = \angle ABD + \angle DBC$. So we get $\angle ABC = 90^o$.

Thus, ΔABC is a right triangle.

And we know ΔDBF is a right triangle, too. Also, we know $\angle F = \angle C$, because of the fact above.

So ΔABC and ΔDBF share the same set of three angles, and thus, are similar.

So we can say that $\frac{FD}{AC} = \frac{BD}{AB}$.

And we know $AC = 2AD$, $BD = AD$, $AB = 3$, and $FD = 8$, since FD is the diameter, which is 8.

So assuming $a = AD$, we get this:

$$\frac{FD}{AC} = \frac{BD}{AB} \Rightarrow \frac{8}{2a} = \frac{a}{3} \Rightarrow \frac{4}{a} = \frac{a}{3} \Rightarrow a^2 = 12 \Rightarrow a = \pm 2\sqrt{3}.$$

And we know $a = AD > 0$, since a is a length. So we get $AD = 2\sqrt{3}$.